Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 2

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is : **[1]**

   a) 8.5 cm b) 13 cm

   c) 12 cm d) $\sqrt{119}$

2. The points A(9, 0), B(9, 6), C(-9, 6) and D(-9, 0) are the vertices of a **[1]**

   a) rhombus b) trapezium

   c) rectangle d) square

3. The point of intersection of the x-axis and y-axis is called **[1]**

   a) ordinate b) abscissa

   c) quardant d) origin

4. The probability that it will rain on a particular day is 0.76. The probability that it will not rain on that day is **[1]**

   a) 0.24 b) 0.76

   c) 0 d) 1

5. The centroid of a triangle divides the median in the ratio **[1]**

   a) 2 : 1 b) 1 : 2

   c) 1 : 3 d) 3 : 1

6. The sum of two numbers is 8. If their sum is four times their difference, then the numbers are **[1]**

a) None of these

b) 7 and 1

c) 6 and 2

d) 5 and 3

7. A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is r, then the volume of the cylinder is **[1]**

a) $2\pi r^3$

b) $8\pi r^3$

c) $\frac{8}{3}\pi r^3$

d) $4\pi r^3$

8. A letter is chosen at random from the word ASSASSINATION. The probability that it is a vowel is **[1]**

a) $\frac{6}{13}$

b) $\frac{7}{13}$

c) $\frac{6}{31}$

d) $\frac{3}{13}$

9. The angry Arjun carried some arrows for fighting with Bheeshma. With half the arrows, he cut down the arrows thrown by Bheeshma on him and with six other arrows he killed the rath driver of Bheeshma. With one arrow each, he knocked down respectively the rath, flag and bow of Bheeshma. Finally, with one more than four times the square root of arrows, he laid Bheeshma unconscious on an arrow bed. The total number of arrows that Arjun had, is **[1]**

a) 100

b) 96

c) 80

d) 120

10. A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will not be a white marble? **[1]**

a) $\frac{1}{3}$

b) $\frac{7}{9}$

c) $\frac{2}{9}$

d) $\frac{4}{9}$

11. If $3x = \text{cosec}\,\theta$ and $\frac{3}{x} = \cot\theta$ then $3\left(x^2 - \frac{1}{x^2}\right) = ?$ **[1]**

a) $\frac{1}{9}$

b) $\frac{1}{81}$

c) $\frac{1}{27}$

d) $\frac{1}{3}$

12. The LCM of $2^3 \times 3^2$ and $2^2 \times 3^3$ **[1]**

a) 2×3^2

b) $2^3 \times 3^3$

c)

d)

$2^2 \times 3^2$ $2^2 \times 3$

13. Which of the following equations has 2 as a root? **[1]**

a) $2x^2 - 7x + 6 = 0$ b) $3x^2 - 6x - 2 = 0$

c) $x^2 + 3x - 12 = 0$ d) $x^2 - 4x + 5 = 0$

14. The coordinates of the point on x -axis which are equidistant from the points (-3, 4) and (2, 5) are **[1]**

a) (-23, 0) b) (20, 0)

c) None of these d) (4/5, 0)

15. If $\sum f_i x_i = 625$ and $\sum f_i = 25$, then the value of $\overline{x}$ is **[1]**

a) 63 b) 64

c) 25 d) 26

16. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of 'm' is **[1]**

a) 3 b) 1

c) 2 d) 4

17. A ladder 12 m long rests against a wall. If it reaches the wall at a height of $6\sqrt{3}$ m, then the angle of elevation is **[1]**

a) 60° b) 30°

c) 75° d) 45°

18. The area of the triangle formed by the lines x = 3, y = 4 and x = y is **[1]**

a) 1 sq. unit b) 1/2 sq. unit

c) None of these d) 2sq. unit

19. **Assertion (A):** In a triangle PQR, A and B are points on sides PQ and PR such that $\frac{PA}{AQ} = \frac{PB}{BR}$, then AB || QR. **[1]**
Reason (R): By converse of BPT.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If p is a prime number then H.C.F. of p, p^2 and p^3 is p. **[1]**
Reason (R): H.C.F. of 3 number is smallest number among them.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the system of equations by using the method of substitution: **[2]**
$3x - 5y = -1$
$x - y = -1$

22. There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of Rs 100 each, 100 of them contain a cash prize of Rs 50 each and 200 of them contain a cash prize of Rs 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize? **[2]**

23. Identify the type of polynomials given below (on the basis of degree): **[2]**

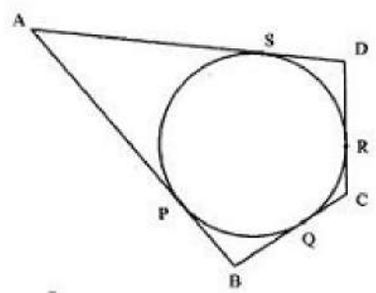
i. $3x^2 + 4x + c$

ii. $3y^3 - 4y^2 + 2y$

iii. $6y + 5$

iv. $3 - \sqrt{2}x^3 + \frac{2}{7}x - 4x^2$

24. A quadrilateral ABCD is drawn to the circumference of a circle. Prove that: AB + CD = AD + BC **[2]**



OR

Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that radius of the circle is 7 cm.

25. Name the type of quadrilateral formed, if any, by the points (4, 5), (7, 6), (4, 3), (1, 2), and give a reason for your answer. **[2]**

OR

If G (-2, 1) is the centroid of a ΔABC and two of its vertices are A(1, -6) and B(-5, 2), find the third vertex of the triangle.

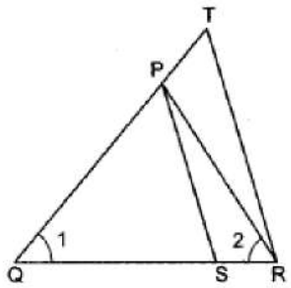
Section C

26. The cost of 4 pens and 4 pencil boxes is Rs.100. Three times the cost of a pen is Rs.15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box. **[3]**

[3]

27. Find the acute angle θ, when $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$.

28. In Fig. if $\frac{QT}{PR}=\frac{QR}{QS}$ and $\angle 1=\angle 2$. Prove that $\Delta PQS\sim\Delta TQR$. **[3]**



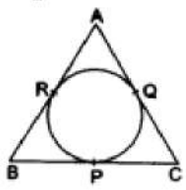
29. Prove $\frac{1}{2+\sqrt{3}}$ is an irrational number. **[3]**

OR

Let d be the HCF of 24 and 36. Find two numbers a and b, such that d = 24a + 36b.

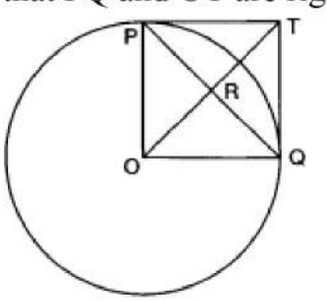
30. The pilot of an aircraft flying horizontally at a speed of 1200 km/hr. observes that the angle of depression of a point on the ground changes from 30° to 45° in 15 seconds. Find the height at which the aircraft is flying. **[3]**

31. In the given figure, the incircle-of $\triangle ABC$ touches the sides BC, CA and AB at P, Q and R respectively. Prove that (AR + BP + CQ) = (AQ + BR + CP) $=\frac{1}{2}$(perimeter of ΔABC). **[3]**



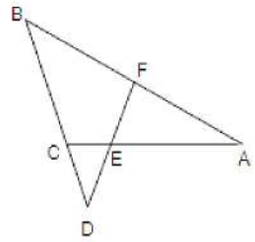
OR

In figure $PO\perp QO$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.



Section D

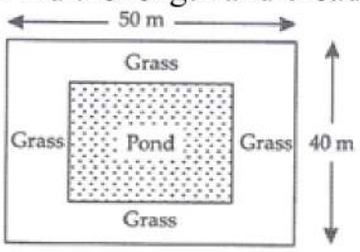
32. In the given figure , a $\angle AEF=\angle AFE$ and E is the mid-point of CA. Prove that $\frac{BD}{CD}=\frac{BF}{CE}$ **[5]**



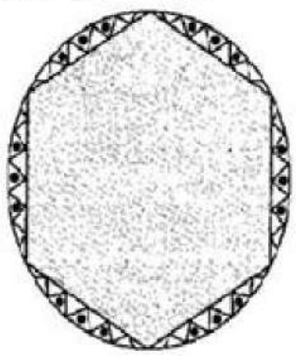
33. A dealer sells a toy for ₹ 24 and gains as much per cent as the cost price of the toy. Then find the cost price of the toy. **[5]**

OR

In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond



34. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (use $\sqrt{3} = 1.7$) **[5]**



OR

A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use $\pi = 3.14$)

i. minor sector

ii. major sector

iii. minor segment

iv. major segment

35. The following table gives weekly wages in rupees of workers in a certain commercial organization. The frequency of class 49-52 is missing. It is known that the mean of the frequency distribution is 47.2. Find the missing frequency. **[5]**

Weekly wages (₹)	40-43	43-46	46-49	49-52	52-55
Number of workers	31	58	60	?	27

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Find the amount saved by Sehaj on 10^{th} day.

(ii) Find the amount saved by Sehaj on 25^{th} day.

OR

Find in how many days Sehaj saves ₹1400.

(iii) Find the total amount saved by Sehaj in 30 days.

37. **Read the text carefully and answer the questions:** **[4]**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.

(i) The volume of the ice-cream without hemispherical end.

(ii) The volume of the ice-cream with a hemispherical end.

(iii) Find the volume her brother ice cream?

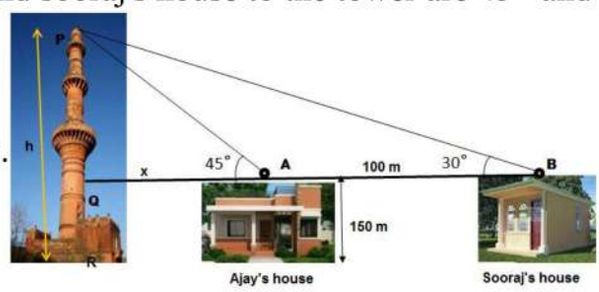
OR

Whose quantity of ice cream is more and by how much?

38. **Read the text carefully and answer the questions:** **[4]**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of

elevation of ajay's house to the tower and sooraj's house to the tower are 45^{o} and 30^{o} respectively as shown in the figure.



(i) Find the height of the tower.

(ii) What is the distance between the tower and the house of Sooraj?

OR

Find the distance between top of tower and top of Ajay's house?

(iii) Find the distance between top of the tower and top of Sooraj's house?

Solution

Section A

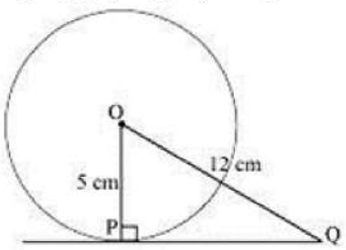
1. **(d)** $\sqrt{119}$

   Explanation:

   We know that the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

   $OP \perp PQ$

   By applying Pythagoras theorem in ΔOPQ,

   $OP^2 + PQ^2 = OQ^2$

   $5^2 + PQ^2 = 12^2$

   $PQ^2 = 144 - 25$

   $PQ = \sqrt{119}$cm

2. **(c)** rectangle

   Explanation: A (9, 0), B(9, 6), C(- 9, 6) and D(- 9, 0) are the given vertices.

   Then,

   $AB^2 = (9 - 9)^2 + (6 - 0)^2$

   $= (0)^2 + (6)^2 = 0 + 36 = 36$ units

   $BC^2 = (- 9 - 9)^2 + (6 - 6)^2$

   $= (-18)^2 + (0)^2 = 324 + 0 = 324$ units

   $CD^2 = (- 9 + 9)^2 + (0 - 6)^2 = (0)^2 + (-6)^2 = 0 + 3 = 36$ units

   $DA^2 = (- 9 - 9)^2 + (0 - 0)^2 = (- 18)^2 + (0)^2 = 324 + 0 = 324$ units

   Therefore, we have:

   $AB^2 = CD^2$ and $BC^2 = DA^2$

   Now, the diagonals are:

   $AC^2 = (- 9 - 9)^2 + (6 - 0)^2 = (- 18)^2 + (6)^2 = 324 + 36 = 360$ units

   $BD^2 = (- 9 - 9)^2 + (0 - 6)^2 = (- 18)^2 + (- 6)^2 = 324 + 36 = 360$ units

   Therefore,

   $AC^2 = BD^2$

   Hence, *ABCD* is a rectangle.

3. **(d)** origin

   Explanation: The point of intersection of the x-axis and y-axis is called an origin.

   The coordinates of the origin are (0, 0).

4. **(a)** 0.24

   Explanation: Given: P (It will rain on a particular day) = 0.76

   ∴ P (It will not rain on a particular day) = 1 - P (It will rain particular day)

   = 1 - 0.76 = 0.24

5. **(a)** 2 : 1
   Explanation: The centroid of a triangle is the centre of the triangle which is the point of intersection of all the three medians of the triangle and divides the median in the ratio 2 : 1 The median is a line drawn from the mid-point of a side to the opposite vertex.
6. **(d)** 5 and 3
   Explanation: $x + y = 8$
   $x = 8 - y$ ... (i)
   $x + y = 4(x - y)$ ... (ii)
   Substitute (i) in (ii)
   $8 = 4x - 4y$
   $2 = x - y$
   $2 = 8 - y - y$
   $2y = 8 - 2$
   $y = 3$
   therefore, $x = 8 - 3 = 5$
   Hence, Numbers are 5 and 3
7. **(a)** $2\pi r^3$
   Explanation: Volume of a sphere = $(4/3)\pi r^3$
   Volume of a cylinder = $\pi r^2 h$
   Given, sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder and the radius of the sphere is *r*.
   Thus, height of the cylinder = diameter = 2r and base radius = r
   Volume of the cylinder = $\pi \times r^2 \times 2r = 2\pi r^3$
8. **(a)** $\frac{6}{13}$
   Explanation: Vowels present in the given word are A, A, I, A, I, O = 6
   Number of possible outcomes = {A, A, I, A, I, O} = 6
   Number of total outcomes = 13
   Required Probability = $\frac{6}{13}$
9. **(a)** 100
   Explanation: Let Arjun had x arrows.
   According to question,
   $\frac{x}{2} + 6 + 3 + 4\sqrt{x} + 1 = x$
   $\Rightarrow 10 + 4\sqrt{x} = \frac{x}{2}$
   $\Rightarrow 20 + 8\sqrt{x} = x$
   $\Rightarrow 8\sqrt{x} = x - 20$
   $\Rightarrow 64x = x^2 - 40x + 400$
   $\Rightarrow x^2 - 104x + 400 = 0$
   $\Rightarrow x^2 - 100x - 4x + 400 = 0$
   $\Rightarrow x(x - 100) - 4(x - 100) = 0$
   $\Rightarrow (x - 100)(x - 4) = 0$
   $\Rightarrow x - 100 = 0$ and $x - 4 = 0$
   $\Rightarrow x = 100$ and $x = 4$ [which is not possible]
   Therefore, Arjun had 100 arrows.
10. **(b)** $\frac{7}{9}$
    Explanation: Total number of marbles = 3 + 2 + 4 = 9.

Number of non-white marbles = 3 + 4 = 7.

∴ P (getting a non-white marble) = $\frac{7}{9}$

11. **(d)** $\frac{1}{3}$

Explanation: $\text{cosec}^2\,\theta - \cot^{2\theta} = 1$

$\implies (3x)^2 - \left(\frac{3}{x}\right)^2 = 1 \Rightarrow 9x^2 - \frac{9}{x^2} = 1 \Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) = \frac{1}{9}$

$\Rightarrow 3\left(x^2 - \frac{1}{x^2}\right) = 3 \times \frac{1}{9} = \frac{1}{3}$

12. **(b)** $2^3 \times 3^3$

Explanation: L.C.M. of $2^3 \times 3^2$ and $2^2 \times 3^3$ is the product of all prime numbers with the greatest power of every given number, hence it will be $2^3 \times 3^3$

13. **(a)** $2x^2 - 7x + 6 = 0$

Explanation: Given, $2x^2 - 7x + 6 = 0$

If 2 satisfies the above equation then 2 is a root.

Now, $2(2)^2 - 7(2) + 6 = 0$

∴ 2 is a root of this equation

14. **(c)** None of these

Explanation: Let the point be A(a, 0) be equidistant from the two given points P(-3, 4) and Q(2, 5) So applying distance formula, we get,

$AP^2 = AQ^2$

Therefore,

$(a + 3)^2 + (-4)^2 = (a - 2)^5 + 5^2$

$10a = 4$

$a = \frac{2}{5}$

Hence the coordinates of A are $\left(\frac{2}{5}, 0\right)$

So the answer is none of these.

15. **(c)** 25

Explanation: $\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{625}{25} = 25$

16. **(c)** 2

Explanation: First, find the HCF of 65 and 117

$117 = 65 \times 1 + 52$

$65 = 52 \times 1 + 13$

$52 = 13 \times 4 + 0$ (zero remainder)

Therefore, HCF (117 , 65) is 13

Now,

∴ $65m - 117 = 13$

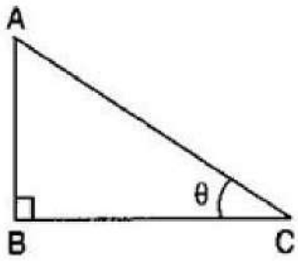
$\Rightarrow 65m = 13 + 117$

$\Rightarrow 65m = 130$

$\Rightarrow m = 2$

17. **(a)** 60°

Explanation: Let the ladder be AC of the length 12 m.



Then the height AB is $6\sqrt{3}$meter.

$\therefore \sin\theta = \frac{AB}{AC}$

$\Rightarrow \sin\theta = \frac{6\sqrt{3}}{12}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

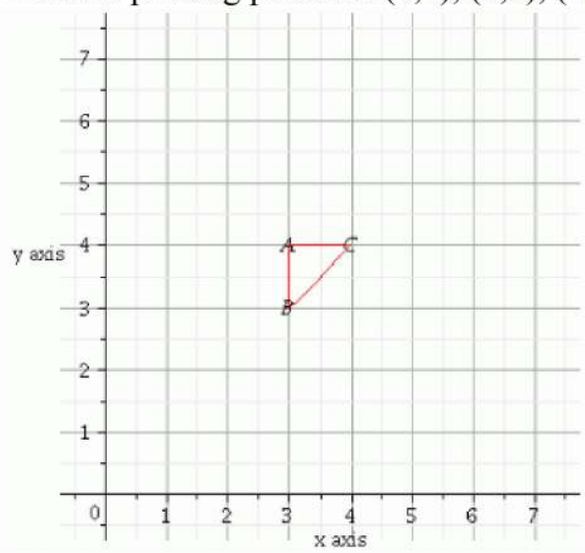
$\Rightarrow \sin\theta = \sin 60°$

$\Rightarrow \theta = 60°$

18. **(b)** 1/2 sq. unit

Explanation: Given x = 3, y = 4 and x = y

We have plotting points as (3,4), (3,3), (4,4) when x = y



Therefore, area of $\triangle ABC = \frac{1}{2}$(Base $\times$ Height) $= \frac{1}{2}$ (AB $\times$ AC) $= \frac{1}{2}$ (1 $\times$ 1) $= \frac{1}{2}$

Area of triangle ABC is $\frac{1}{2}$ square units.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Applicability of converse of Basic Proportionality Theorem, i.e., of Thale's theorem.

20. **(c)** A is true but R is false.

Explanation: A is true but R is false.

Section B

21. The given system of equations is:

$3x - 5y = -1$ .....(i)

$x - y = -1$ .....(ii)

From (ii), we get

$y = x + 1$

Substituting, $y = x + 1$ in (i), we get

$3x - 5(x + 1) = -1$

$\Rightarrow -2x - 5 = -1$

$\Rightarrow x = -2$

Putting x = -2 in y = x + 1 we get y = -1.

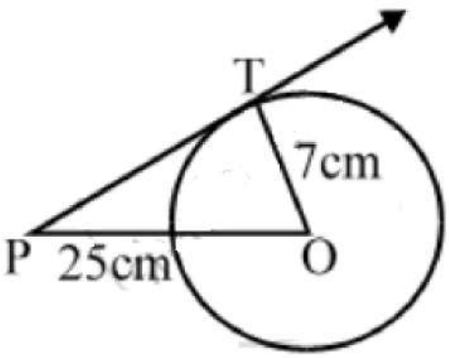
Hence, the solution of the given system of equations is x = -2 and y = -1.

22. Total number of envelopes $= 1000$
Let A = Event that envelope contains no cash
Number of envelopes containing no cash $= 1000 - (10 + 100 + 200) = 690$
$\therefore$ P(A) $= \frac{690}{100} = \frac{69}{100} = 0.69$

23. i. Here, the highest power of x in the given polynomial is 2, so it is a quadratic polynomial.
ii. Here, the highest power of y in the given polynomial is 3, so it is a cubic polynomial.
iii. Here, the highest power of y in the given polynomial is 1, so it is a linear polynomial.
iv. Here, the highest power of x in the given polynomial is 3, so it is a cubic polynomial.

24. Let the sides of the quadrilateral ABCD touch the circle at P, Q, R and S. Since, the lengths of the tangents from an external point to a given circle are equal.
$\therefore$ AP = AS
$\Rightarrow$ BP = BQ
CR = CQ
$\Rightarrow$ DR = DS
Adding, (AP + BP) + (CR + DR) = (BQ + CQ) + (AS + DS)
$\Rightarrow$ AB + CD = BC + AD.
Hence proved

OR



Let O is the centre of the circle and P is a point such that OP = 25 cm and PT is the tangent to the circle.
$OT =$ radius $= 7cm$
In $\triangle$OTP, we have $\angle$T = 90°
$OP^2 = OT^2 + PT^2$ [By using Pythagorean theorem]
$\Rightarrow (25)^2 = 7^2 + PT^2 \Rightarrow PT^2 = 625 - 49 = 576$
$\Rightarrow PT = 24cm$

25. (4, 5), (7, 6), (4, 3), (1, 2)
Let A $\rightarrow$ (4, 5), B $\rightarrow$ (7, 6), C $\rightarrow$ (4, 3) and D $\rightarrow$ (1, 2)
Then, $AB = \sqrt{(7-4)^2 + (6-5)^2}$
$= \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$
$BC = \sqrt{(4-7)^2 + (3-6)^2}$
$\sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$
$CD = \sqrt{(1-4)^2 + (2-3)^2}$
$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$
$DA = \sqrt{(4-1)^2 + (5-2)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(4-4)^2 + (3-5)^2} = 2$

$BD = \sqrt{(1-7)^2 + (2-6)^2}$

$= \sqrt{36+16} = \sqrt{52}$

We see that

AB = CD, opposite sides are equal

BC = DA

and AC $\neq$ BD ..... Diagonals are unequal

Hence, the quadrilateral ABCD is a parallelogram.

OR

If G (-2, 1) is the centroid of a ΔABC and two of its vertices are A(1, -6) and B(-5, 2).

Let the third vertex be C(a, b)

Then, the co-ordinates of its centroid are

$G\left(\frac{1-5+a}{3}, \frac{-6+2+b}{3}\right)$ i.e $G\left(\frac{-4+a}{3}, \frac{-4+b}{3}\right)$

But given that the centroid is G(-2, 1)

$\frac{-4+a}{3} = -2$ and $\frac{-4+b}{3} = 1$

-4 + a = -6 and -4 + b =3

a = -2 and b = 7

Hence, the third vertex C of ΔABC is (-2, 7)

Section C

26. Let us suppose that the cost of pens be Rs.x

let price of pencil boxes = Rs.y

According to question

4x + 4y = 100 .....(1)

3x - y = 15 .....(2)

Multiplying equation (2) by 4, we get

4(3x - y)= 15 × 4

12x - 4y = 60.....(3)

Adding equation (1) and (3), we get

4x + 4y + 12x - 4y = 100+ 60

16x = 160

x = Rs. 10

Substituting the value of x in equation (1), we get

4(10) + 4y = 100

40 + 4y = 100

4y = 60

So, y = Rs. 15

So, price of pens be = Rs. 10

And price of pencil boxes = Rs. 15

27. According to question

$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \quad \frac{(\cos\theta - \sin\theta) + (\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta) - (\cos\theta + \sin\theta)} = \frac{(1-\sqrt{3}) + (1+\sqrt{3})}{(1-\sqrt{3}) - (1+\sqrt{3})}$ [*Applying componendo and dividendo*]

$\Rightarrow \quad \frac{2\cos\theta}{-2\sin\theta} = \frac{2}{-2\sqrt{3}}$

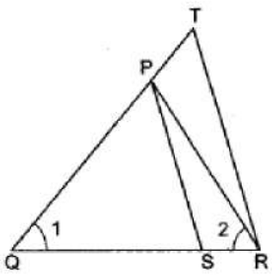
$\Rightarrow \quad \cot\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

28. According to the question,we are given that,

$\frac{QT}{PR} = \frac{QR}{QS}$ [Given]

$\Rightarrow \quad \frac{QT}{QR} = \frac{PR}{QS}$ ...(i)

We also have,



$\angle 1 = \angle 2$ [Given]

$\Rightarrow$ PR = PQ [Sides opposite to equal angles are equal] ...(ii)

From (i) and (ii), we get

$\frac{QT}{QR} = \frac{PQ}{QS}$

$\Rightarrow \quad \frac{PQ}{QT} = \frac{QS}{QR}$ ...(iii)

Thus, in triangles PQS and TQR, we have

$\frac{PQ}{QT} = \frac{QS}{QR}$ and $\angle$PQS = $\angle$TQR = $\angle$Q

So, by SAS-criterion of similarity, we obtain $\Delta PQS \sim \triangle TQR$.

29. Let $\frac{1}{2+\sqrt{3}}$ be a rational number.

A rational number can be written in the form of $\frac{p}{q}$ where p,q are integers.

$\frac{1}{2+\sqrt{3}} = \frac{p}{q}$

$\Rightarrow \sqrt{3} = \frac{q-2p}{p}$

p, q are integers then $\frac{q-2p}{p}$ is a rational number.

Then$\sqrt{3}$ is also a rational number.

But this contradicts the fact as $\sqrt{3}$ is an irrational number.

So, our supposition is false.

Therefore, $\frac{1}{2+\sqrt{3}}$ is an irrational number.

OR

Let d be the HCF of 24 and 36.

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

HCF = $2^2 \times 3 = 12$

$\Rightarrow d = 12$

Now $d = 24a + 36b$

$12 = 24a + 36b$

When $a = -1$ and $b = 1$, we get

$12 = 24\times(-1) + 36\times(1)$

$= -24 + 36$

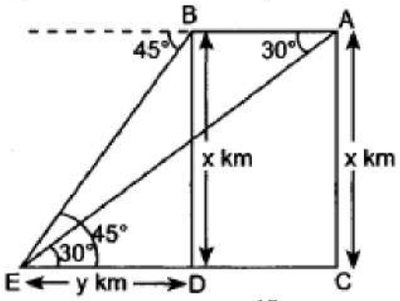
$12 = 12$

So, a = -1 , b = 1 satisfy the equation d = 24a + 36b

$\therefore$ One possible value of a and b is -1 and 1.

30. Distance covered in 15 seconds $= AB$

Speed $= 1200\ km/hr.$



$\therefore$ AB $= 1200 \times \frac{15}{3600} = 5\text{km}$

$AB = DC = 5km$

Let height = x km

In rt. $\triangle$BDE,

$\frac{BD}{ED} = \tan 45° \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y$

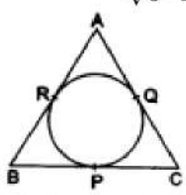
In rt. $\triangle$ACE,

$\frac{AC}{EC} = \tan 30° \Rightarrow \frac{x}{y+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{x}{x+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}x = x + 5 \Rightarrow (\sqrt{3} - 1)x = 5$

$\therefore \quad x = \frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{2} = 6.83\text{km}$

31. 

We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ AR = AQ, ... (i) [tangents from A]

BP = BR, ... (ii) [tangents from B]

CQ = CP .... (iii) [tangents from C]

$\therefore$ (AR + BP + CQ) = (AQ + BR + CP) = k (say).

Perimeter of $\triangle ABC$ = (AB + BC + CA)

= (AR + BR) + (BP + CP) + (CQ+AQ)

= (AR + BP + CQ) + (AQ + BR + CP)

= (k + k) = 2k

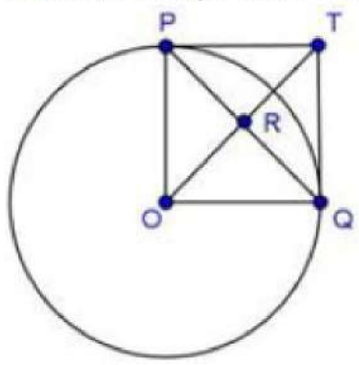
$\Rightarrow \quad k = \frac{1}{2}(\text{ perimeter of } \triangle ABC)$.

$\therefore$ (AR + BP +CQ) = (AQ + BR + CP)

$= \frac{1}{2}(\text{ perimeter of } \triangle ABC)$.

OR

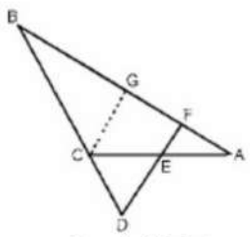
Given, PO$\perp$QO and The tangents to the circle at P and Q intersect at a point T.



Consider, $\Delta TPO \text{ and } \Delta TQO$
PT = TQ [$\because$ Tangets from external point are equal in length]
OT = OT [Common]
$\angle TPO = \angle TQO = 90^\circ$
So, by RHS rule, we have
$\Delta TPO \cong \Delta TQO$
$\Rightarrow \angle PTO = \angle QTO$ ...(i) [C.P.C.T.]
Now, In ΔPTR and ΔQTR
PT = TQ [$\because$ Tangents from external point are equal in length]
$\angle PTO = \angle QTO$ [By equation (i)]
TR = TR [Common]
So, by SAS rule, we have
$\Delta PTR \cong \Delta QTR$
$\therefore$ PR = RQ ...(ii)
And, $\angle TRP = \angle TRQ$
But, $\angle TRP + \angle TRQ = 180^\circ$
$\Rightarrow 2\angle TRP = 180^\circ$
$\Rightarrow \angle TRP = 90^\circ$ ...(iii)
Therefore, PQ and OT are right bisectors of each other.

Section D

32. Given, $\angle AEF = \angle AFE$ and E is the mid-point of CA.
To prove, $\frac{BD}{CD} = \frac{BF}{CE}$
Construction Draw a line CG parallel to DF$(CG||DF)$.



Proof : $\angle AEF = \angle AFE$ and E is the mid-point of CA
$\therefore CE = AE = \frac{AC}{2}$ ...(i)
In $\triangle BDF$, $CG||DF$
By Basic proportionality theorem,
$\frac{BD}{CD} = \frac{BF}{GF}$ ... (ii)
In ΔAFE,
$\angle AEF = \angle AFE$ [$\because$given]
$\Rightarrow AF = AE$ [[$\because$Since, sides opposite to equal angles are equal]
$\Rightarrow AF = AE = CE$ [[$\because$From Eq(i)] ....(iii)
In ΔACG, E is the midpoint of AC and$EF||CG$,
$\therefore FG = AF$ [$\because AE = CE$] ...(iv)
From Eq(ii), Eq(iii) and Eq(iv),
$\frac{BD}{CD} = \frac{BF}{GF}$
$\frac{BD}{CD} = \frac{BF}{CE}$ [$\because GF = AF = CE$]
Hence proved.

33. According to question a dealer sells a toy for Rs. 24 and gains as much per cent as the cost price of the toy.
Let the cost price of the toy be Rs. x.
Then, Gain = x%

$\Rightarrow$ Gain = Rs. $\left(x \times \frac{x}{100}\right)$ = Rs. $\frac{x^2}{100}$.

Therefore, S.P. = C.P. + Gain = $x + \frac{x^2}{100}$.

But, S.P. =Rs. 24.

$\therefore \quad x + \frac{x^2}{100} = 24$ [Given]

$\Rightarrow 100x + x^2 = 2400$

$\Rightarrow x^2 + 100x - 2400 = 0$

$\Rightarrow x^2 + 120x - 20x - 2400 = 0$

$\Rightarrow \quad x(x + 120) - 20(x + 120) = 0$

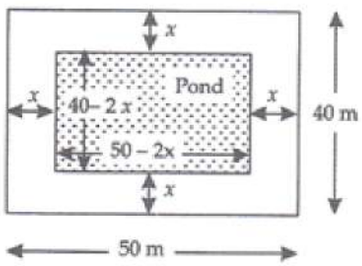
$\Rightarrow (x + 120)(x - 20) = 0$

$\Rightarrow x = 20, -120$

$\Rightarrow x = 20$ [because $x > 0$]

Hence, the cost price of the toy is Rs. 20.

OR



Let width of the pond be x m. Then,

The length of pond = $(50 - 2x)m$ and the breadth of pond = $(40 - 2x)m$

Area of grass around the pond = 1184 m^2

$\Rightarrow$ Area of Lawn - Area of Pond = 1184

$\Rightarrow 50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow 2000 - (2000 - 100x - 80x + 4x^2) - 1184 = 0$

$\Rightarrow 2000 - (2000 - 180x + 4x^2) - 1184 = 0$

$\Rightarrow 2000 - 2000 + 180x - 4x^2 - 1184 = 0$

$\Rightarrow 4x^2 - 180x + 1184 = 0$

$\Rightarrow 4(x^2 - 45x + 296) = 0$

$\Rightarrow x^2 - 45x + 296 = 0$

Factorise now,

$\Rightarrow x^2 - 37x - 8x + 296 = 0$

$\Rightarrow x(x - 37) - 8(x - 37) = 0$

$\Rightarrow (x - 37)(x - 8) = 0$

$\Rightarrow x - 37 = 0$ or $x - 8 = 0$

$\Rightarrow x = 37$ or $x = 8$

When x = 37, then

The length of pond = $50 - 2 \times 37$

= 50 - 74

= -24 m (Length cannot be negative)

When x = 8, then

The length of pond = 50 – 2x

= $50 - 2 \times 8$

= 50 - 16

= 34 m

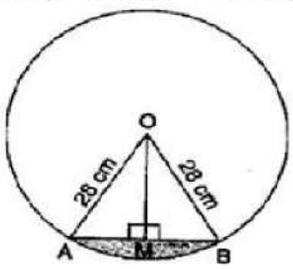
And the breadth of the pond
= 40 - 2x
= 40 - 2 × 8
= 40 - 16
= 24 m
Therefore, the length and breadth of the pond are 34 m and 24 m respectively.

34. r = 28 cm and $\theta = \frac{360}{6} = 60^\circ$

Area of minor sector = $\frac{\theta}{360}\pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$

= 410.67 cm^2

For, Area of △AOB,



Draw OM ⊥ AB.
In right triangles OMA and OMB,
OA = OB [Radii of same circle]
OM = OM [Common]
∴ △OMA ≅ OMB [RHS congruency]
∴ AM = BM [By CPCT]

⇒ AM = BM = $\frac{1}{2}$AB and ∠AOM = ∠BOM = $\frac{1}{2}$AOB = $\frac{1}{2} \times 60^o = 30^o$

In right angled triangle OMA, $\cos 30^o = \frac{OM}{OA}$

⇒ $\frac{\sqrt{3}}{2} = \frac{OM}{28}$

⇒ OM = $14\sqrt{3}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA}$

⇒ $\frac{1}{2} = \frac{AM}{28}$

⇒ AM = 14 cm

⇒ 2 AM = 2 × 14 = 28 cm

⇒ AB = 28 cm

∴ Area of △AOB = $\frac{1}{2}$ × AB × OM = $\frac{1}{2} \times 28 \times 14\sqrt{3} = 196\sqrt{3} = 196 \times 1.7 = 333.2$ cm 2

∴ Area of minor segment = Area of minor sector - Area of △AOB

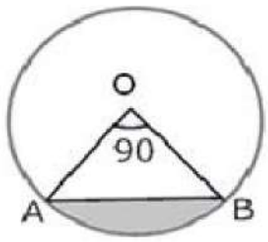
= 410.67 - 333.2 = 77.47 cm^2

∴ Area of one design = 77.47 cm^2

∴ Area of six designs = 77.47 × 6 = 464.82 cm^2

Cost of making designs = 464.82 × 0.35 = Rs. 162.68

OR



i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

$= 78.50 = 78.5 \text{ cm}^2$

ii. Area of major sector = Area of circle - Area of minor sector

$= \pi(10)^2 - \frac{90}{360}\pi(10)^2 = 3.14\ (100) - \frac{1}{4}(3.14)\ (100)$

$= 314 - 78.50 = 235.5 \text{ cm}^2$

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle \text{OAB} = \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\ (\because \angle AOB = 90^\circ)$

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{1}{4}(3.14)\ (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 \text{ cm}^2$

iv. Area of major segment = Area of the circle - Area of minor segment

$= \pi(10)^2 - 28.5$

$= 100(3.14) - 28.5$

$= 314 - 28.\ 5 = 285.5 \text{ cm}^2$

35. Let the missing frequency be f the assumed mean be A = 47 and h = 3.

Calculation of Mean

Class-Intervals	**mid-values x_i**	**f_i**	**$d_i = x_i - 47.5$**	$u_i = \frac{x_i - 47.5}{3}$	**$f_i u_i$**
40-43	41.5	31	-6	-2	-62
43-46	44.5	58	-3	-1	-58
46-49	47.5	60	0	0	0
49-52	50.5	f	3	1	f
52-55	53.5	27	6	2	54
	$N = \sum f_i = 176 + f$				**$\Sigma f_i u_i = f - 66$**

We have,

$\overline{X} = 47.2$, A = 47.5 and h = 3

$\therefore \overline{X} = A + h\left\{\frac{1}{N}\Sigma f_i u_i\right\}$

$\Rightarrow 47.2 = 47.5 + 3 \times \left\{\frac{f-66}{176+f}\right\}$

$- 0.3 = 3 \times \left\{\frac{f-66}{176+f}\right\}$

$\Rightarrow \frac{-1}{10} = \frac{f-66}{176+f} \Rightarrow -176 - f = 10f - 660 \Rightarrow 11f = 484 \Rightarrow f = 44$

Hence, the missing frequency is 44

Section E

36. **Read the text carefully and answer the questions:**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Money saved on 1st day = ₹27.5

$\because$ Sehaj increases his saving by a fixed amount of ₹2.5

$\therefore$ His saving form an AP with a = 27.5 and d = 2.5

$\therefore$ Money saved on 10th day,

$a_{10} = a + 9d = 27.5 + 9(2.5)$

$= 27.5 + 22.5 = ₹50$

(ii) $a_{25} = a = 24d$

$= 27.5 + 24(2.5)$

$= 27.5 + 60 = ₹\ 87.5$

OR

Let $S_n = 387.5$, a = 27.5 and d = 2.5

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$

$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$

$\Rightarrow 775 = 55n + 2.5n^2 - 2.5n$

$\Rightarrow 25n^2 + 525n = 7750 = 0$

$\Rightarrow n^2 + 21n - 310 = 0$

$\Rightarrow (n + 31)(n - 10) = 0$

$\Rightarrow n = -31$ reject $n = 10$ accept

So in 10 years Sehaj saves ₹ 387.5.

(iii)Total amount saved by Sehaj in 30 days.

$= \frac{30}{2}[2 \times 27.5 + (30-1) \times 2.5]$

$= 15(55 + 29(2.5)$

$= ₹1912.5$

37. **Read the text carefully and answer the questions:**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice

cream with length 9 cm, width 4cm and thickness 2 cm.

(i) For cone, radius of the base (r) = 2.5cm $= \frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$

For hemisphere,

Radius (r) = 2.5cm $= \frac{5}{2}$cm

$\therefore$ Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

The volume of the ice-cream without hemispherical end = Volume of the cone

$= \frac{825}{14}\text{cm}^3$

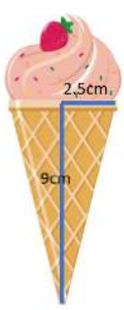
(ii) For cone, radius of the base (r) = 2.5cm $= \frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = 2.5cm $= \frac{5}{2}$cm

$\therefore$ Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$

$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

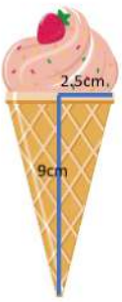
(iii)For cone, Radius of the base (r) = 2.5cm = $\frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

= $\frac{825}{14}$cm^3



For hemisphere,

Radius (r) = 2.5cm = $\frac{5}{2}$cm

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}$cm^3

Volume of rectangular brick shaped ice cream = 9 × 4 × 2 = 72 cm^3

OR

For cone, Radius of the base (r)

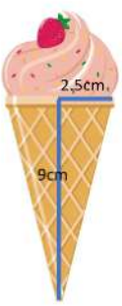
= 2.5cm = $\frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

= $\frac{825}{14}$cm^3



For hemisphere,

Radius (r) = 2.5cm = $\frac{5}{2}$cm

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}$cm^3

Sheetal ice cream quantity is more than her brother

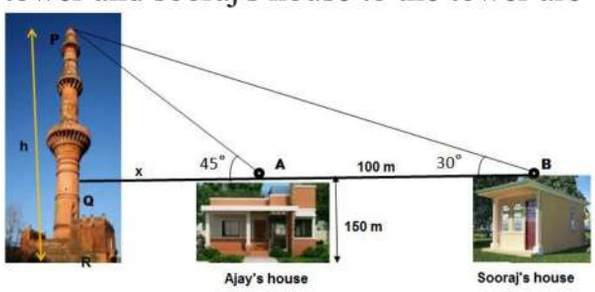
Volume of Sheeta's ice cream - Volume her brother's ice cream

= 91.66 - 72 = 19.66 cm^3

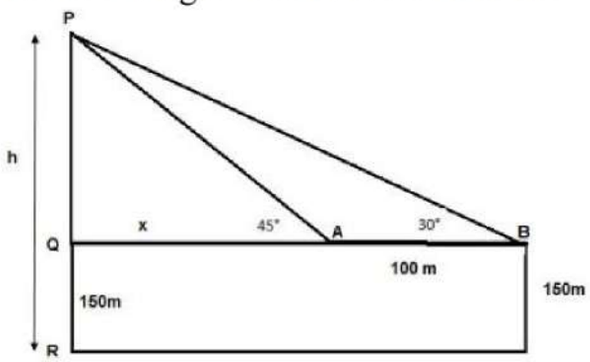
38. **Read the text carefully and answer the questions:**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the

tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.



(i) The above figure can be redrawn as shown below:



Let PQ = y

In △PQA,

$\tan 45 = \frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y ...(i)

In △PQB,

$\tan 30 = \frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

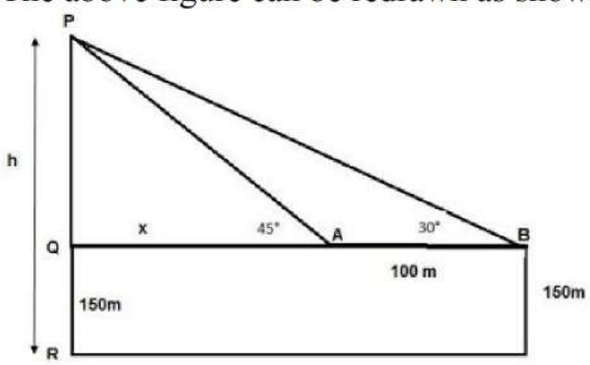
$x = \frac{100}{\sqrt{3}-1} = 136.61$ m

From the figure, height of tower h = PQ + QR

= x + 150 = 136.61 + 150

h = 286.61 m

(ii) The above figure can be redrawn as shown below:

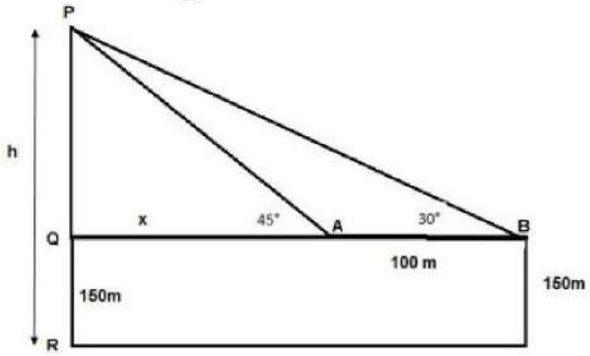


Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

OR

The above figure can be redrawn as shown below:



Distance between top of the tower and top of Ajay's house is PA

In $\triangle$PQA

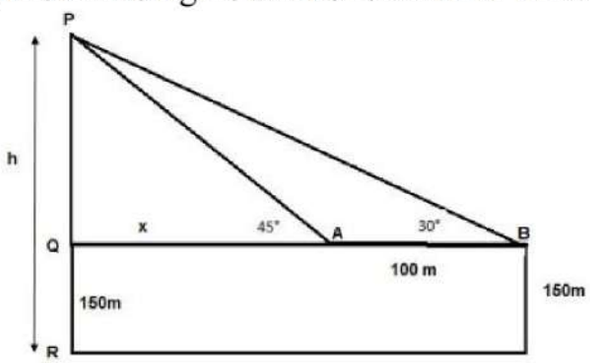
$\sin 45^{o} = \frac{PQ}{PA}$

$\Rightarrow PA = \frac{PQ}{\sin 45^{o}}$

$\Rightarrow \text{PA} = \frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$

$\Rightarrow$ PA = 193.20 m

(iii)The above figure can be redrawn as shown below:



Distance between top of tower and Top of Sooraj's house is PB

In $\triangle$PQB

$\sin 30^{o} = \frac{PQ}{PB}$

$\Rightarrow PB = \frac{PQ}{\sin 30^{o}}$

$\Rightarrow \text{PB} = \frac{y}{\frac{1}{2}} = 2 \times 136.61$

$\Rightarrow$ PB = 273.20 m